COMPUTATIONAL PHYSICS (PH881)

MINI PROJECT

REFRACTION OF LIGHT THROUGH ATMOSPHERE

Model based on Wave theory and Corpuscular theory

Submitted to:

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ABSTRACT

The objective of this project is to model the bending of light during transition between media with different index without using Snell's law [sin(i)/ sin(r) = n] and plot trajectory of light through the layers of atmosphere. This project is inspired by two theories - Wave theory by Christian Huygens and Corpuscular theory by Newton. Numerical methods like Bisection Algorithm and Newton-Raphson method have been compared for their suitableness. Plots of the trajectory of light through atmosphere derived from Wave theory and Corpuscular theory are compared and analysed.

CONTENTS

1. INTRODUCTION

1.1 Wave theory of light - Huygens' Principle

Huygens' Principle states that:

Every point on a wave-front may be considered a source of secondary spherical wavelets which spread out in the forward direction at the speed of light. The new wave-front is the tangential surface to all of these secondary wavelets. [2]

According to Huygens' principle, a plane light wave propagates through free space at the speed of light, c. The light rays associated with this wave-front propagate in straight-lines, as shown in Fig 1. It is also fairly straightforward to account for the laws of reflection and refraction using Huygens' principle.

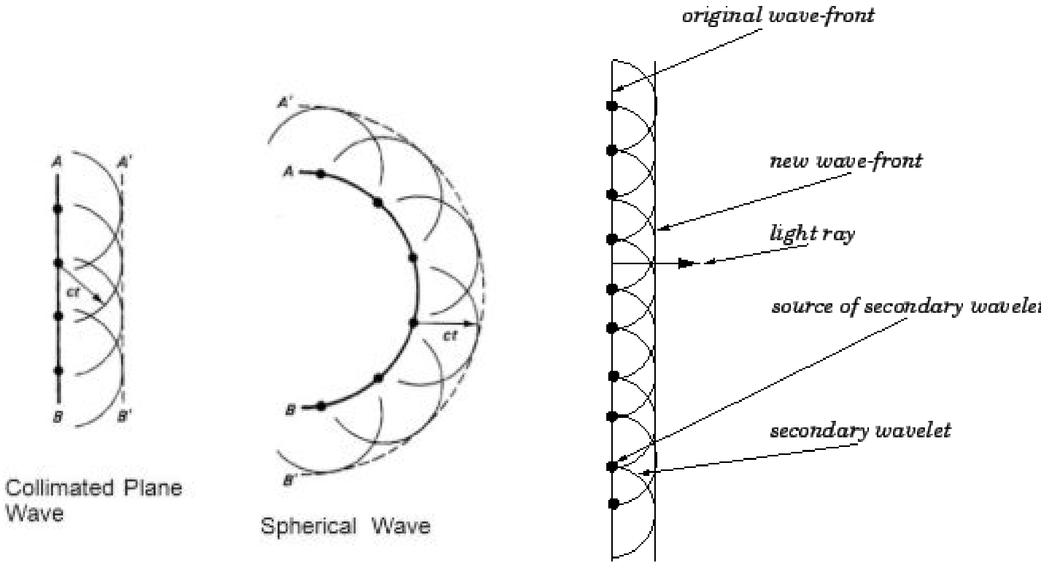


Fig 1. Huygens' Principle [2]

1.2 Corpuscular theory of light

The paper in American Journal of Physics (85(12):921-925) titled 'Newton's Atmospheric theory of refraction' by Michael Nauenberg presents the Newton's Corpuscular theory of light. Newton assumed that light consisted of particles (corpuscles), and derived Snell's refraction law from the assumption that the velocity of these corpuscles is proportional to the index of refraction of the medium, and the acceleration is proportional to its gradient. All further calculations of angular momentum and angle of deviation are based on these assumptions. However, we know that the velocity of light decreases upon travelling to a medium of higher refractive index. So, it is assumed that velocity is inversely proportional to the index of refraction of the medium. Angular momentum of the particles remains conserved for all the multiple deflections occurring through the layers of the atmosphere. The formulae in this paper have been reformulated considering this modification. It is also stated that the decimal part of the refractive index, (whole number part being 1) of the atmosphere increases exponentially as one moves towards the earth.

2. MATHEMATICAL MODEL

2.1 Corpuscular theory

With reference to fig. 2, assuming unit mass of light corpuscle, Angular momentum l is given by,

$$l = r * v * sin(\xi) \quad \{1\}$$

where r - distance from centre of earth

v - speed of light

ξ - Angle of refraction

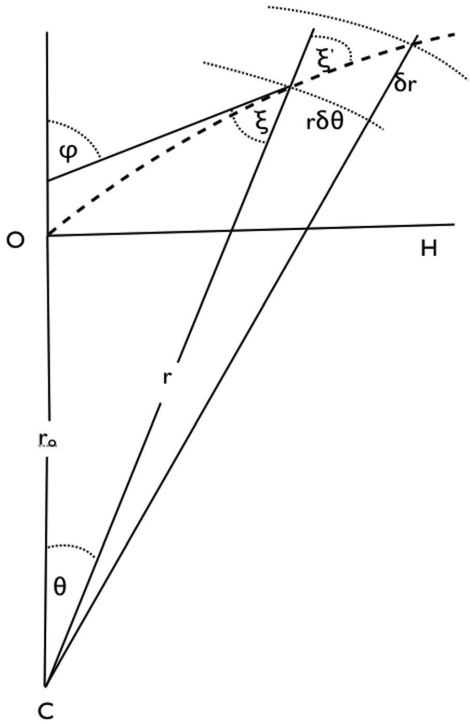


Fig 2. Diagram showing the variables in atmospheric refraction

With our assumption that, v is inversely proportional to refractive index n, {1} can be re-written as,

$$l = r * sin(\xi)/ n$$

$$dl = dr * sin(\xi)/ n - (r/n^2) * dn/dr * sin(\xi) + r/n * cos (\xi) * d\xi/dr \quad \{2\}$$

Upon simplification by equating dl = 0,

$$d\xi/dr = - tan(\xi) * (1/r - dn/ (n * dr) \quad \{3\}$$

From Fig. 2,

$$\phi = \theta + \xi \quad \{4\}$$

{3} is written as,

$$(\xi_{i+1} - \xi_i) /dr = - tan(\xi_i) * (1/r - dn/ (n * dr) \quad \{5\}$$

$$n = 1 + K * e^{(-h/H)} \quad \{6\}$$

n - refractive index
K - constant found by Newton = 0.003161
h - distance above the earth’s surface
H = max. height upto which refractive index is not negligible = 10km

2. METHODOLOGY

2.1 Corpuscular theory

{6} is substituted in {5} and solved iteratively with initial condition,

$$\xi_0 = \phi_0$$

$\phi(0)$ **=** Observation angle (Final Angle of approach of light to the earth's surface)

$$x_{i+1} = x_i + dr.\,sin(\xi_i)$$
$$y_{i+1} = y_i + dr.\,cos(\xi_i)$$

2.2 Wave theory

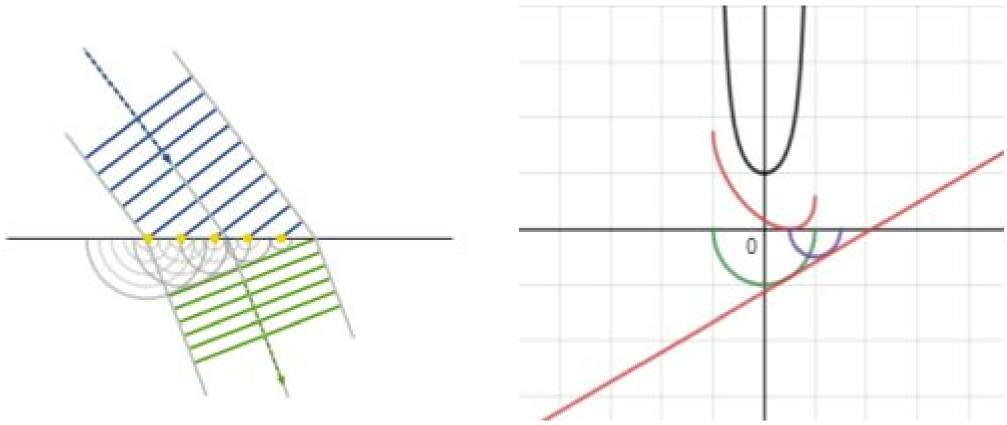


Fig 3(a). Wavefronts giving rise to secondary wavelets followed by refraction [3]
Fig 3(b). 2 Wavelets, tangent, Function(x) touching the x axis at the point of tangency

- 3 collinear points constitute the wavefront (direction of light propagation)
- They emanate circular wavelets which increase in radius once they strike the interface
- Tangent is drawn from the 3rd point to the wavelets arising out of the 2 points which struck the interface initially
- The 3 collinear points move perpendicular to the tangent

Finding the points of tangency

$$Function(x) = y(circle) - y(tangent)$$
$$y(circle) = -\sqrt{r^2 - x^2}$$
$$y(tangent) = m * x - r * \sqrt{1 + m^2}$$

m = slope of tangent = $width * sin(\xi) / \sqrt{(2 * n)^2 - (width * sin(\xi))^2}$

$width$ = thickness of light beam , r = radius of circular wavelet

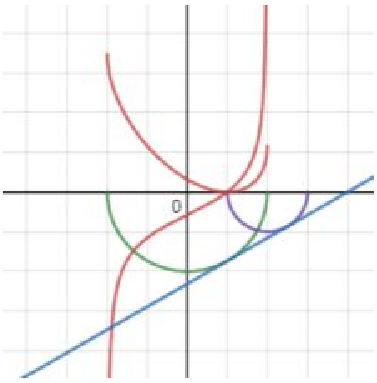
Suitableness of numerical methods

- Methods like bisection which work on the basis of sign change can't be used since tangent is always on one side of the circle and doesn't intersect it.
- At the point of tangency, not only the Function is zero, but its derivative as well .So,

$$Function\,for\;computation \;=\; d/dx(Function(x))$$

$$Derivative\,function \;=\; d^2/dx^2(Function(x)$$

- When the light beam strikes an interface whose co-ordinates are away from initial origin (0,0) , points of tangency are found considering (0,0) as the centre of circle. The tangent is then translated to the new frame.



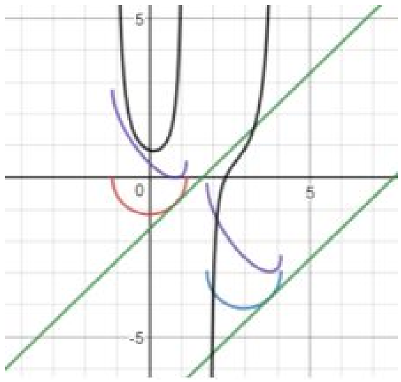


Fig 4(a). *d/dx(Function(x))* intersecting x axis at the point of tangency [4]
Fig 4(b). Parallel translation of tangent and circles to origin for simplified computation

4. RESULTS AND CONCLUSIONS

Trajectory of light is plotted using both the methods

4.1 Corpuscular theory

Parameters:
Observation angle, ϕ_0 = 1^0 with respect to the normal
Maximum height, H = 10km
Newton's constant, K = 0.003161
Incremental increase in distance, dr = 0.1km
Radius of Earth's surface, ro = 6366 km

4.2 Wave theory

The atmospheric layers are discretized on the basis of the refractive index. They are spaced equally. Screenshot of the animation is presented.

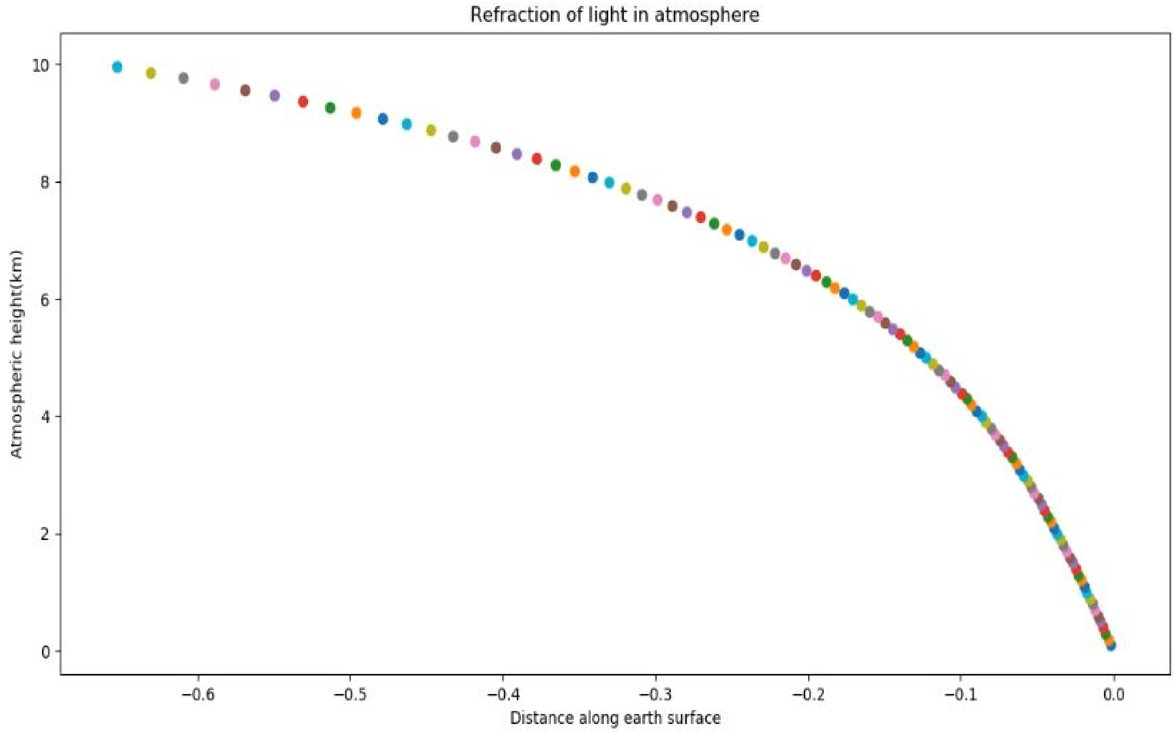


Fig 5. Trajectory of light particle through atmosphere by Corpuscular theory

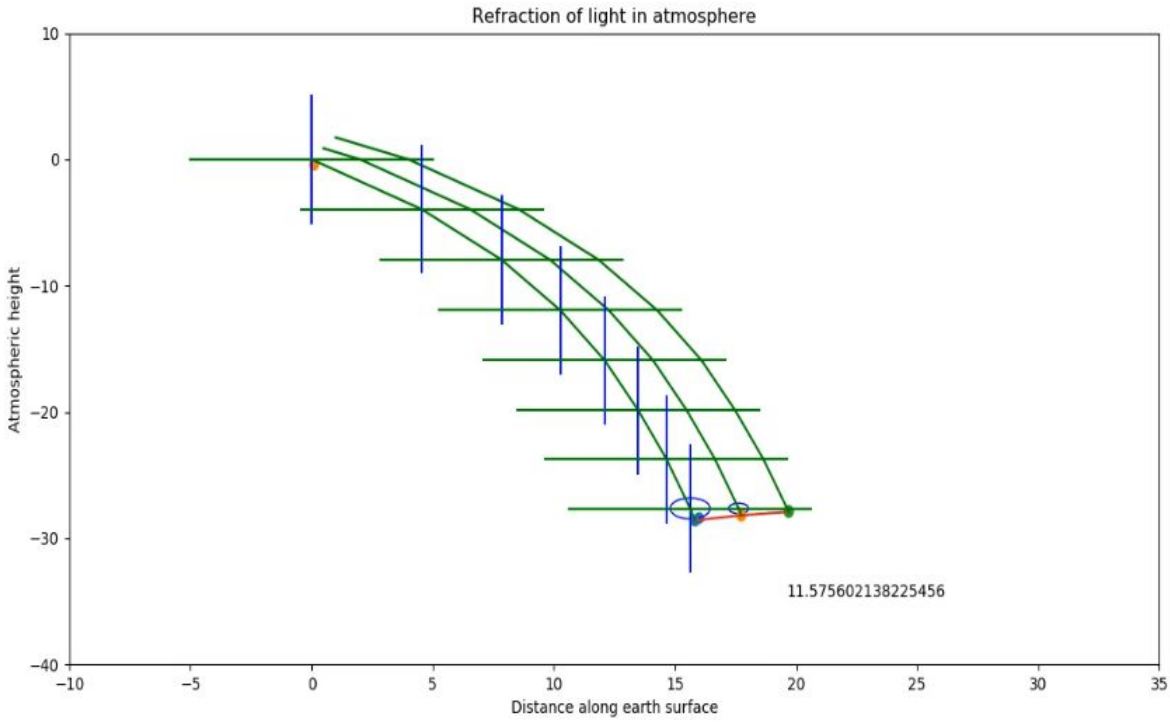


Fig 6. Trajectory of light particle through atmosphere by Wave theory
(Screenshot of the simulation)

Observations:

The trajectories are similar to y = $\sqrt{-x}$, saturating as they move away from origin. The reduced spacing between dots in Fig 5 shows the decrease of velocity i.e., increase of refractive index as one move towards earth.
This also shows that the decimal part of the refractive index decreases exponentially with increasing atmospheric height.

5. REFERENCES

[1] Michael Nauenberg, *Newton's atmospheric refraction of light,* American Journal of Physics **85**, 921 (2017); https://doi.org/10.1119/1.5009672
[2] http://farside.ph.utexas.edu/teaching/302l/lectures/node150.html
[3] https://www.thoughtco.com/huygens-principle-2699047
[4] https://www.desmos.com/calculator

Please find the code for the project here:
https://github.com/PrajwalBBharadwaj/Refraction-through-atmosphere